New Regular Black Holes: Geometry, Matter Sources and Shadow Profiles

Anjan Kar

IIT Kharagpur

anjankar.phys@gmail.com

Based on : ***A. Kar and S. Kar, "Novel regular black holes: geometry, source and shadow", arxiv: 2308.12155***

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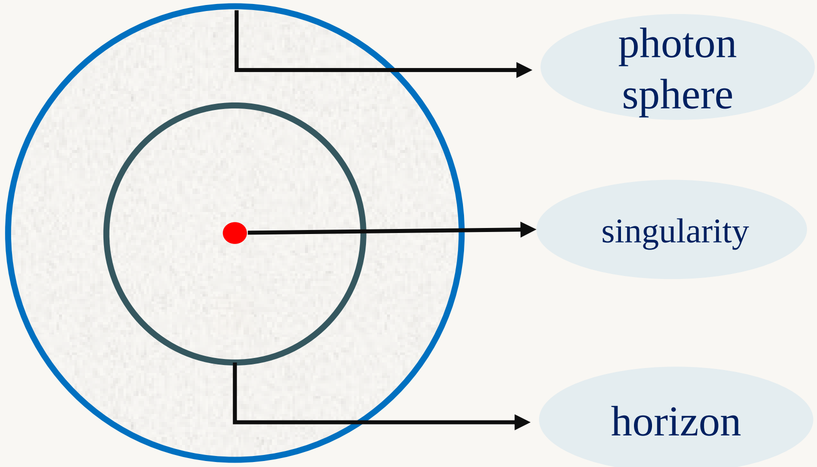
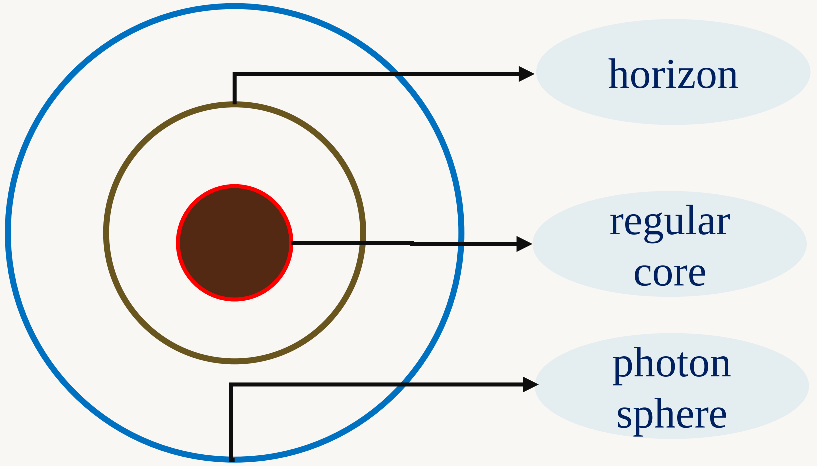
What is a Regular Black Hole?

Singularity is inevitable in general relativity. → *Break down of classical GR* → *Regular black hole may be a substitute*

- Characteristics of a regular black hole:
  I. *Singularity inside horizon replaced by a regular core*
  II. *Other characteristic surfaces exist*
  III. *Can be generated from Einstein equation in presence of matter*
- Regular metric can be checked through:
  I. *Regularity of scalar invariants*
  II. *Geodesic completeness*



Schwarzschild BH



A typical regular BH

Some Notable Proposal of Regular Black Holes

J. M. Bardeen, "Non-singular general-relativistic gravitational collapse", in Proceedings of International Conference GR5, 1968, Tbilisi, USSR

$$ds^2 = -\left(1 - \frac{2Mr^2}{\left(r^2+(2Ml^2)^{2/3}\right)^{3/2}}\right)dt^2 + \left(1 - \frac{2Mr^2}{\left(r^2+(2Ml^2)^{2/3}\right)^{3/2}}\right)^{-1} dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)$$

Bardeen regular BH

S. A. Hayward, "Formation and evaporation of regular black holes," Phys. Rev. Lett. 96 (2006) 031103

$$ds^2 = -\left(1 - \frac{2Mr^2}{r^3+2Ml^2}\right)dt^2 + \left(1 - \frac{2Mr^2}{r^3+2Ml^2}\right)^{-1} dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)$$

Hayward regular BH

$l^2 = 0, \quad \Rightarrow \quad -g_{tt} = g^{rr} = 1 - \frac{2M}{r}, \quad \Rightarrow$ ***Standard Schwarzschild solution***

A New Proposal of Regular Geometry and Features

A. Kar and S. Kar, "Novel regular black holes: geometry, source and shadow", arxiv: 2308.12155

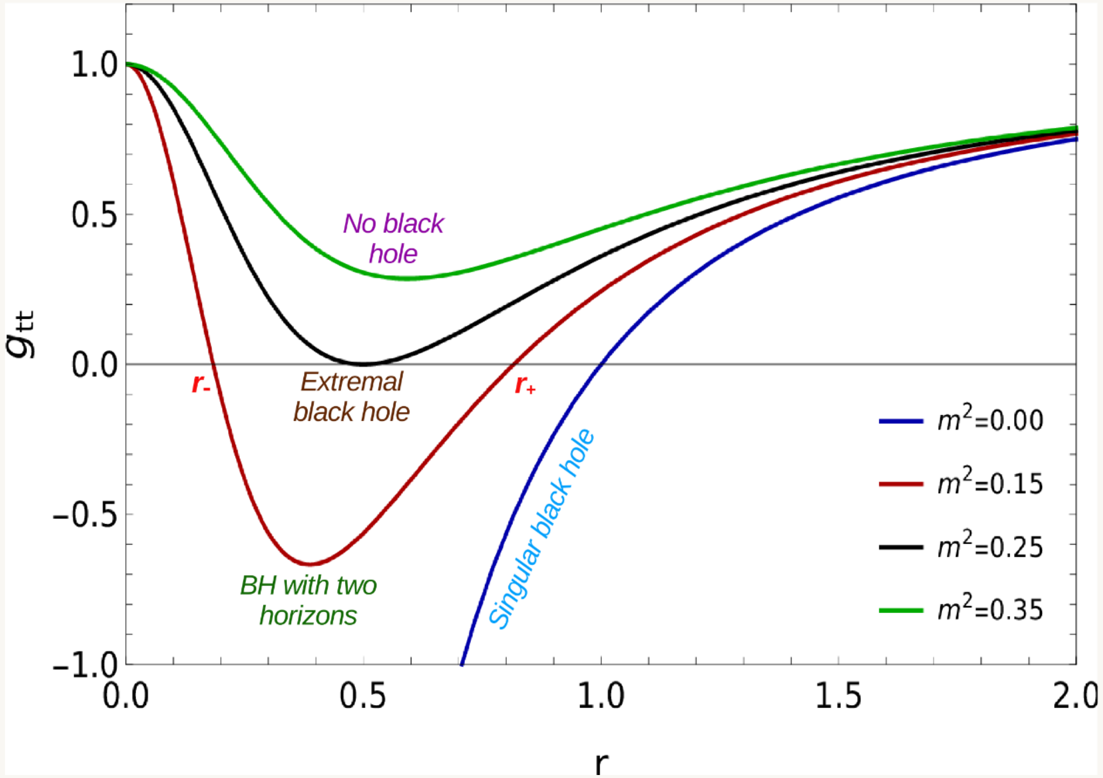
$$ds^2 = -\left(1 - \frac{b_0^2\, r^2}{\left(r^2+g^2\right)^2}\right) dt^2 + \left(1 - \frac{b_0^2\, r^2}{\left(r^2+g^2\right)^2}\right)^{-1} dr^2 + r^2\left(d\theta^2 + \, d\phi^2\right)$$

- g^2= regularization parameter; $g^2 \;=\; 0, \Rightarrow$ *Mutated Reissner-Nordstrom Geometry* $(M = 0)$ *with imaginary charge*

$$ds^2 = -\left(1 - \frac{b_0^2}{r^2}\right) dt^2 + \left(1 - \frac{b_0^2}{r^2}\right)^{-1} dr^2 + r^2\left(d\theta^2 + \, d\phi^2\right)$$

- $r \to 0, \quad -g_{tt} = g^{rr} \approx \left(1 - \frac{b_0^2}{g^4} r^2\right) \;\Rightarrow$ **a de-Sitter core** and $r \to \infty$ **asymptotically flat**

Horizon location:



$$-g_{tt} = f(r) = \left(1 - \frac{b_0^2\, r^2}{(r^2+m^2 b_0^2)^2}\right); \qquad \boldsymbol{g^2 = m^2 b_0^2}$$

Horizon at, $-g_{tt} = f(r) = 0$;

$$\boldsymbol{(r^2 + m^2 b_0^2)^2 - b_0^2 r^2 = 0}$$

- $m^2 = 0$ ⇨ ***singular RN-type solution***, *horizon at* $r = b_0$
- $0 < m^2 < 0.25$ ⇨ ***a family of regular BH*** *having two horizons*
- $m^2 = 0.25$ ⇨ ***extremal regular BH*** *with single horizon*
- $m^2 > 0.25$ ⇨ ***a regular space time without horizon***

Proof of the regularity

1. Curvature invariant:

- Ricci scalar: $R = \frac{12b_0^2\left(g^4 - g^2r^2\right)}{\left(r^2+g^2\right)^2};$

- Ricci contraction: $R_{\mu\nu}R^{\mu\nu} = \frac{4b_0^4\left(9g^8 - 18g^6r^2 + 34g^4r^4 - 10g^2r^6 + r^8\right)}{\left(r^2+g^2\right)^8};$

- Kretschmann scalar: $K = \frac{8b_0^4\left(3g^8 - 6g^6r^2 + 34g^4r^4 - 22g^2r^6 + 7r^8\right)}{\left(r^2+g^2\right)^8}$

$r \to 0$ all the curvature scalars reaches a finite value

2. Completeness of all causal geodesics:

- Radial timelike geodesics:

$$g_{tt}\dot{t}^2 + g_{rr}\dot{r}^2 = -1$$

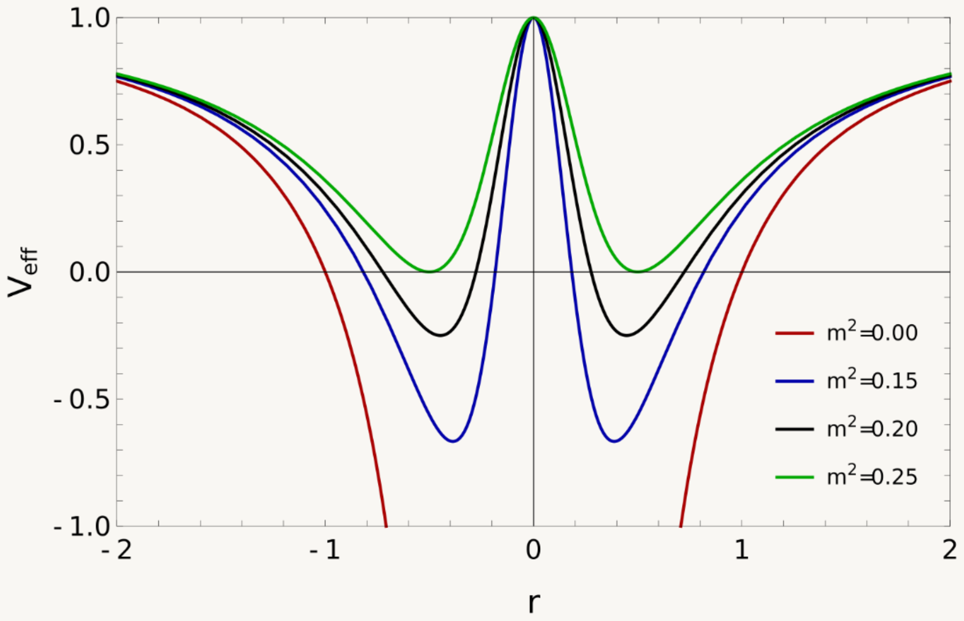
- Conserved quantity; $E = -g_{tt}\dot{t}$; $\dot{r}^2 = E^2 - 1 + \frac{b_0^2 r^2}{(r^2+g^2)^2} = E^2 - V_{eff}(r)$

$$V_{eff}(r) = -g_{tt} = 1 - \frac{b_0^2 r^2}{(r^2+m^2 b_0^2)^2}$$

- Affine parameter: $\lambda(r) = \int \frac{dr}{\sqrt{\dot{r}^2}}$

- Everywhere finite behaviour of $-g_{tt}$ in the entire domain of 'r'

  from $-\infty$ to $+\infty$ ⇨ ***a Geodesically complete spacetime***



For more details *T. Zhou and L. Modesto, "Geodesic incompleteness of some popular regular black holes", Phys. Rev. D 107, 044016 (2023)*

Energy-momentum tensors and energy conditions:

- Einstein's equation $G^{\alpha}_{\beta} = \kappa T^{\alpha}_{\beta}$,

$$T^t_t = T^r_r = -\frac{(3g^2 - r^2)b_0^2}{\kappa(r^2+g^2)^3} = \frac{b_0^2 r^2}{\kappa(r^2+g^2)^3} - 3g^2\frac{b_0^2}{\kappa(r^2+g^2)^3}$$

$$T^\theta_\theta = T^\phi_\phi = -\frac{(3g^4 - 8g^2r^2 + r^4)b_0^2}{\kappa(r^2+g^2)^4} = -\frac{r^2(r^2-2g^2)b_0^2}{\kappa(r^2+g^2)^4} \underbrace{- 3g^2\frac{(g^2-2r^2)b_0^2}{\kappa(r^2+g^2)^4}}_{\textit{Satisfies NEC \& WEC}}$$

Violates NEC & WEC

Satisfies NEC & WEC

- Now, total energy-momentum tensors:

Energy conditions	Range of validation
NEC	$r^2 \leq 5g^2$
WEC	$r^2 \leq 3g^2$
SEC	$(4-\sqrt{13})g^2 \leq r^2 \leq (4+\sqrt{13})g^2$
DEC	$r^2 \leq g^2$

Matter sources of the geometry

1. *Nonlinear electrodynamics:*

$$S = \int d^4x \sqrt{-g}\,\left(R - L_1(F) - L_2(F)\right);$$

☒ NEC & WEC ☑

$$F = F_{\alpha\beta}F^{\alpha\beta},$$

$$F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha$$

- Energy-momentum tensor corresponding to $L_1(F)$ and $L_2(F)$

$$T^{(i)}{}_{\mu\nu} = 2\left(\frac{\partial L_{(i)}}{\partial F} F_{\mu\alpha}F^{\alpha}{}_{\nu} - \frac{1}{4} g_{\mu\nu}\, L_{(i)}(F)\right); \quad i = 1,2$$

- Covariant equation of motions:

$$G_{\alpha\beta} = \kappa \sum_i T^{(i)}{}_{\alpha\beta}; \qquad \nabla_\mu\left(\frac{\partial L_{(i)}}{\partial F} F^{\mu\nu}\right) = 0$$

- We choose following forms of $L_1(F)$ and $L_2(F)$:

$$\mathbf{L_1(F) = -\frac{\gamma\, F}{\left(1+\eta\sqrt{F}\right)^3}} \;\; \& \;\; \mathbf{L_2(F) = \frac{3\,\gamma\,\eta\, F^{3/2}}{\left(1+\eta\sqrt{F}\right)^3}}$$

- As $\mathrm{T_t^t = T_r^r}$ and $\mathrm{T_\theta^\theta = T_\phi^\phi}$ ⇨ $\mathbf{F = 2F_{tr}F^{tr} + 2F_{\theta\phi}F^{\theta\phi}}$; only $\mathrm{F_{tr} \neq 0}$, $\mathrm{F_{\theta\phi} \neq 0}$

- A magnetic solution with $F_{\theta\phi} = -q\sin\theta$, and $\gamma = \frac{b_0^2}{\kappa q^2}$, $\eta = \frac{g^2}{\sqrt{2q^2}}$:

$$\boxed{\textit{Satisfy EOM;} \quad G_{\alpha\beta} = \kappa \sum_i T^{(i)}{}_{\alpha\beta}}$$

- So, total Lagrangian density: $\mathbf{L(F)} = \boldsymbol{L_1(F) + L_2(F)} = \mathbf{-\frac{\gamma F\left(1-3\eta\sqrt{F}\right)}{\left(1+\eta\sqrt{F}\right)^3}}$

- Note, $g^2 = 0$ or, $\eta^2 = 0$ ⇨ *Singular RN-type solution*

$\mathbf{L(F) = -\gamma F}$ linear electrodynamics *Expected*

- Nonlinear Maxwell-like equations for a given $L(F)$;

$$\nabla_\mu F^{\mu\upsilon} + \left(\frac{\partial^2 L}{\partial F^2} \Big/ \frac{\partial L}{\partial F}\right)\left(\partial_\mu F\right) F^{\mu\upsilon} = J_e^\upsilon \quad \& \quad \nabla_\mu \tilde{F}^{\mu\upsilon} = J_m^\upsilon$$

- Here, $F_{\theta\phi} = -q\sin\theta$; therefore $J_e^\upsilon = \{0,0,0,0\}$ and $J_m^\upsilon = \{q\delta(r), 0,0,0\}$

A nonlinear magnetic monopole at $r = 0$ supports our geometry

2. Brane-world gravity:

- A new kind of decomposition of energy-momentum tensor:

$$T_t^t = T_r^r = \frac{b_0^2 r^2}{\kappa(r^2+g^2)^3} - \frac{3g^2 b_0^2}{\kappa(r^2+g^2)^3}$$

$$T_\theta^\theta = T_\phi^\phi = -\frac{b_0^2 r^2}{\kappa(r^2+g^2)^3} \underbrace{- 3g^2 b_0^2 \frac{(g^2-3r^2)}{\kappa(r^2+g^2)^4}}_{\text{extra matter, satisfies NEC,WEC}}$$

tracefree part

- Einstein equation on the four-dimensional 3-brane: *R. Maartens and K. Koyama, Living Rev. Relativ.* ***13****, 5 (2010)*

$$G_{\alpha\beta} = -\Lambda g_{\alpha\beta} + \kappa T_{\alpha\beta} + 6\frac{\kappa}{\lambda}\underbrace{S_{\alpha\beta}}_{\text{Quadratic stress energy on the brane}} - \mathcal{E}_{\alpha\beta}$$

On-brane matter

Traceless geometric quantity controlled by extra dimension

- A large brane tension λ and a zero Λ leads:

$$G_{\alpha\beta} = \kappa T_{\alpha\beta} - \mathcal{E}_{\alpha\beta}$$

- $-\mathcal{E}_{\alpha\beta}$ model the *trace free part* of energy-momentum tensors

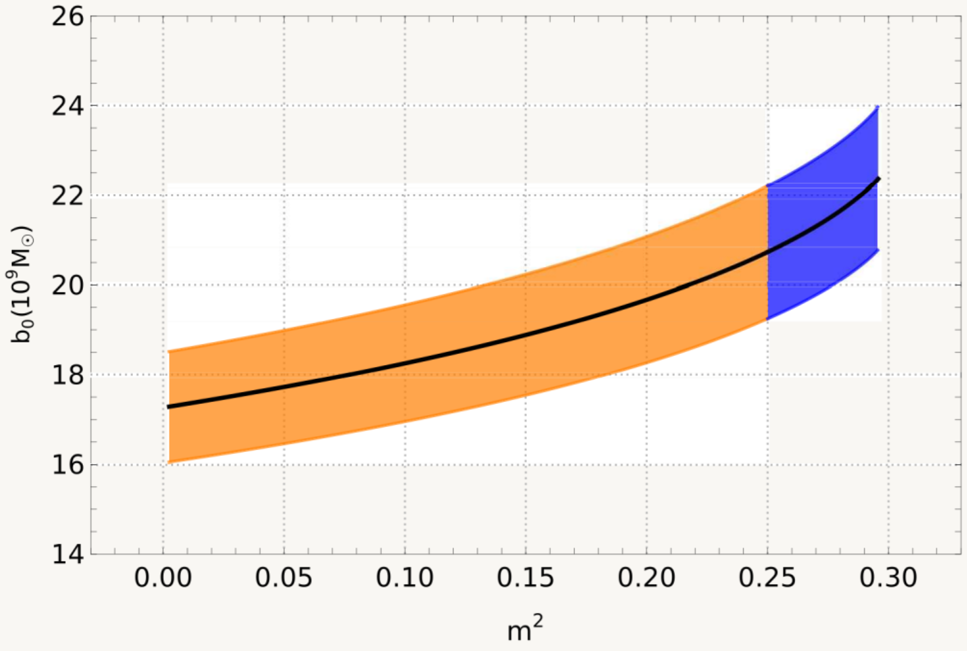
- *Therefore, a suitable matter on the brane satisfies NEC & WEC and can model the spacetime.*

Shadow radius of our RBH and observational constraints

- The circular shadow profile of our static regular black hole:

$$r_{sh}^2 = \frac{r_{ph}^2\left(r_{ph}^2 + g^2\right)^2}{\left(r_{ph}^2 + g^2\right)^2 - r_{ph}^2\, b_0^2}; \qquad g^2 = m^2 b_0^2$$

- Observed angular diameter of $M87^*$ is $\phi = (42 \pm 3)\ \mu as$; and distance from observer $(16.8 \pm 0.8)\ Mpc$



- Parameter values corresponding to $\phi = 42\ \mu as$
- Parameter values for RBH
- Parameter values for horizonless compact object

Summary of the constraints on b_0 from shadow of $M87^*$ and $Sgr\ A^*$

Massive object	Distance from the observer	Angular diameter of the shadow (μas)	Constraints on b_0 for regular BH	Unit of b_0
M87*	16.8 Mpc *Steller population measurement* *M. Cantiello et al., Astrophys. J. Lett. 854 (2018)*	42 ± 3 *	$16.05 < b_0 \leq 22.22$	$10^9 M_\odot$
		41.5 ± 0.6 **	$16.83 < b_0 \leq 20.78$	
$Sgr\ A^*$	*8277 pc* *Gravity Collaboration* *R. Abuter et al., Astron. Astrophys. 657, L12 (2022)*	48.7 ± 7 *** *(Shadow diameter)*	$8.46 < b_0 \leq 13.55$	$10^6 M_\odot$
		51.8 ± 2.3 *** *(ring diameter)*	$10.03 < b_0 \leq 13.16$	
	7935 pc (*Keck team*) *T. Do et al., Science 365 no.6454, (2019)*	48.7 ± 7	$8.11 < b_0 \leq 12.99$	
		51.8 ± 2.3	$9.62 < b_0 \leq 12.61$	

* *K. Akiyama et al., Astrophys. J. Lett. 875, L1 (2019)*

** *L. Medeiros et al., Astrophys. J. Lett. 947, L7 (2023)*

*** *K. Akiyama et al., Astrophys. J. Lett. 930, L12 (2022)*

Summary

- A new regular black hole metric is proposed :

$$ds^2 = -\left(1 - \frac{b_0^2\, r^2}{\left(r^2+g^2\right)^2}\right) dt^2 + \left(1 - \frac{b_0^2\, r^2}{\left(r^2+g^2\right)^2}\right)^{-1} dr^2 + r^2\left(d\theta^2 + \, d\phi^2\right)$$

- The family of regular black holes seem to satisfy all the energy conditions in a specific range of r.
- The source is discussed as a nonlinear magnetic monopole, as well as in the context of Brane-world gravity.
- The shadow profile of the RBH is studied and compared with observational results.

Thank You