New regular solutions using a generic density profile

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Singular spacetime

- The vacuum solution of Einstein equation $G_{\mu\nu} = 0$:

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \frac{dr^2}{1 - \frac{2M}{r}} + r^2 \left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

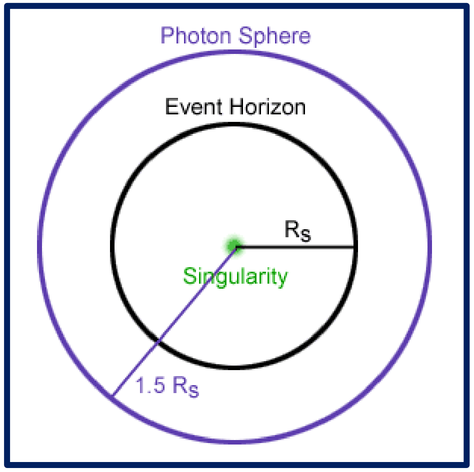
Schwarzschild spacetime

- The event horizon is at $r = 2M$ and $r = 3M$ represents the photon sphere radius.

- Kretschmann scalar: $R_{\mu\nu\lambda\delta}R^{\mu\nu\lambda\delta} = \frac{48M^2}{r^6}$

Divergence of Kretschmann scalar at r = 0, defined as spacetime singularity

- Ways to define spacetime singularity:
  - Infinite value of one curvature scalar
  - Geodesic incompleteness (discuss later)



R. M. Wald, General Relativity. Chicago Univ. Pr., Chicago, USA, 1984

Issues and possible solutions

❑ What makes singularity problematic?

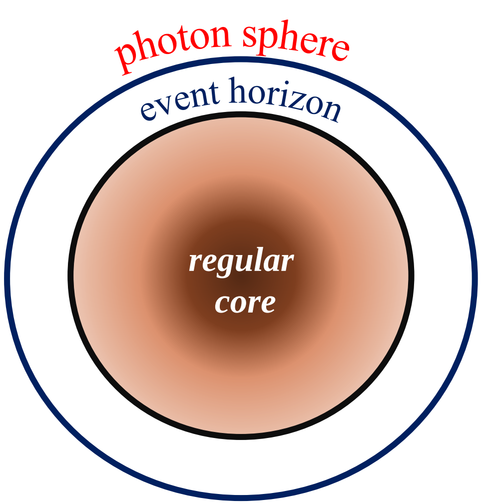
- ➢ Singular spacetimes are not physical.
- ➢ Physical quantities cannot be defined at the point of singularity.
- ➢ Fate of gravitational collapse of initial stable structure is unknown.

❑ Methods for dealing with the issue.

- ➢ Quantum gravity.
- ➢ Nonsingular solutions: wormholes, regular black holes, other exotic compact objects.

❑ Regular black holes:

- ➢ Sakharov* and Gliner** suggested that a de-Sitter core could replace the singularity.
- ➢ Can be constructed from Einstein equation in presence of matter.



* A. D. Sakharov, Zh. Eksp. Teor. Fiz. 49, 345 (1966) [Sov. Phys. JETP 22, 241 (1966)]

** E. B. Gliner, Sov. Phys. JETP 22, 378 (1966).

Several proposals of regular BH

- **Bardeen regular black hole:**

$$ds^2 = -\left(1 - \frac{2Mr^2}{(r^2+g^2)^{\frac{3}{2}}}\right)dt^2 + \left(1 - \frac{2Mr^2}{(r^2+g^2)^{\frac{3}{2}}}\right)^{-1}dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

J. M. Bardeen, in Proceedings of International Conference GR5 , 1968, Tbilisi, USSR.

- **Hayward regular black hole:**

$$ds^2 = -\left(1 - \frac{2Mr^2}{r^3+g^3}\right)dt^2 + \left(1 - \frac{2Mr^2}{r^3+g^3}\right)^{-1}dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

S. A. Hayward, PRL 96 031103 (2006).

- g is *regularization parameter*, at $r \to 0$ de-Sitter space $\left(f(r) \approx 1 - c^2 r^2\right)$ and asymptotically flat.

- **Generalization of Bardeen and Hayward metric:**

$$ds^2 = -\left(1 - \frac{2Mr^{p-1}}{(r^q+g^q)^{\frac{p}{q}}}\right)dt^2 + \left(1 - \frac{2Mr^{p-1}}{(r^q+g^q)^{\frac{p}{q}}}\right)^{-1}dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

J.C.S. Neves, A. Saa, PLB 734, 44 (2014).

Matter sources of these geometries

- Ayon-Beato and Garcia propose the matter as *nonlinear electrodynamics* (NLE),

$$S = \int d^4x\sqrt{-g}\left(\frac{R}{\kappa} + L(F)\right)$$

E. Ayon-Beato, A. Garcia, PRL 80, 5056 (1998).

- Magnetic *monopole* ansatz, $F_{\theta\varphi} = -q_m \sin\theta \quad \Rightarrow \quad F = \frac{1}{4}F^{\mu\rho}F_{\mu\rho} = \frac{{q_m}^2}{2r^4}$

	Bardeen's BH	Hayward's BH
$L(F)$	$\dfrac{-\left(\frac{g^4}{2q_m^2}F\right)^{5/4}}{g^2\left(1+\frac{g^2}{\sqrt{2q_m^2}}\sqrt{F}\right)^{5/2}}$	$\dfrac{-\left(\frac{g^4}{2q_m^2}F\right)^{3/2}}{g^2\left(1+\left(\frac{g^4}{2q_m^2}F\right)^{3/4}\right)^2}$

All of them are constructed via reverse engineering

- Regular black holes in presence of *scalar field* is also available.

K. A. Bronnikov, J. C Fabris, PRD 96, 251101 (2006), K. A Bronnikov, Particles 1, 56 (2018).

- There are *alternate methods* to construct regular black holes.

J. Ovalle, R. Casadio, A. Giusti, PLB 844, 138085 (2023).

Brief Outline

- A method to construct regular solutions
- Two specific regular solutions constructed from the method
  - Regular black holes
  - Regular defect solutions
- An alternative approach to construct the solutions
- Applications of the solutions

Reference: A. Kar, S. Kar, arXiv:2504.12042

The method

- Einstein equation: $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \mathrm{R} = 8\pi T_{\mu\nu}$

- Assumption of spacetime:

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) \qquad f(r) = 1 - \frac{2m(r)}{r}$$

- Assumption of energy-momentum tensor:

$$T^{\mu}_{\nu} = \begin{pmatrix} -\rho & 0 & 0 & 0 \\ 0 & \tau & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}$$

- From Einstein equation we have:

$$\rho = -\tau = \frac{2m'}{8\pi r^2}, \qquad p = -\frac{m''}{8\pi r}$$

- Our approach: Choose $\rho(r)$ ⇒ Derive $m(r)$ ⇒ Obtain τ, p

How to choose the density profile?

$$\rho = \frac{\rho_0 \left(\frac{r}{R}\right)^{\mu-3}}{\left(1+\left(\frac{r}{R}\right)^{\nu}\right)^{\frac{\mu+\alpha}{\nu}}}$$

Parametrized Dekel-Zhao dark matter density profile

H. Zhao, MNRS 278, 488 (1996).

Some known dark matter profiles:

	NFW Astrophys. J. 462, 563 (1996).	Pseudo isothermal MNRS 249, 523 (1991).	King Astron. J. 67, 471 (1962).
Parameter values	$\mu=2, \nu=1, \alpha=0$	$\mu=3, \nu=2, \alpha=-1$	$\mu=3, \nu=2, \alpha=0$
Density profile	$\frac{\rho_0 R}{r}\left(1+\frac{r}{R}\right)^{-2}$	$\rho_0\left(1+\frac{r^2}{R^2}\right)^{-3/2}$	$\rho_0\left(1+\frac{r^2}{R^2}\right)^{-1}$

Regularity of the independent curvature scalars:

E. Zakhary, C. B. G. Mcintosh, GRG 29, 539 (1997).

- Ricci scalar and Ricci contraction:

$$g_{\mu\nu}R^{\mu\nu} = 8\pi(4\rho + r\rho'), \qquad R_{\mu\nu}R^{\mu\nu} = 32\pi^2\left(8\rho^2 + 4r\rho\rho' + r^2\rho'^2\right)$$

$$\mu \geq 3, \quad \nu > 0, \quad \alpha > -3$$

- Kretschmann scalar:

$$R_{\mu\nu\lambda\delta}R^{\mu\nu\lambda\delta} = \frac{48m^2}{r^6} + \frac{64\pi m}{r^3}(-2\rho + r\rho') + 64\pi^2(4\rho^2 + r^2\rho'^2)$$

A new regular black hole

- We consider the *King dark matter density* profile $(\mu = 3, \nu = 2, \alpha = 0)$

$$\rho(r) = \frac{\rho_0}{\left(1 + \frac{r^2}{R^2}\right)^{3/2}}$$

I. King, Astron. J. 67, 471 (1962).

- Solving Einstein equation, we have the corresponding *metric function*:

$$f(r) = 1 + \frac{8\pi\rho_0 R^3}{\sqrt{r^2 + R^2}} + \frac{8\pi\rho_0 R^3}{r} \ln\left(\frac{\sqrt{r^2 + R^2} - r}{R}\right)$$

- At small values of r, $r \to 0$, $f(r) \approx \left(1 - c^2 r^2\right) \Rightarrow$ *a de-Sitter core*

- The *asymptotic expansion* of the metric function:

$$f(r) = 1 + 8\pi\rho_0 R^2 \left[\frac{R}{r} - \frac{\ln(2r/R)}{r/R}\right] + O\left(\frac{1}{r^3}\right)$$

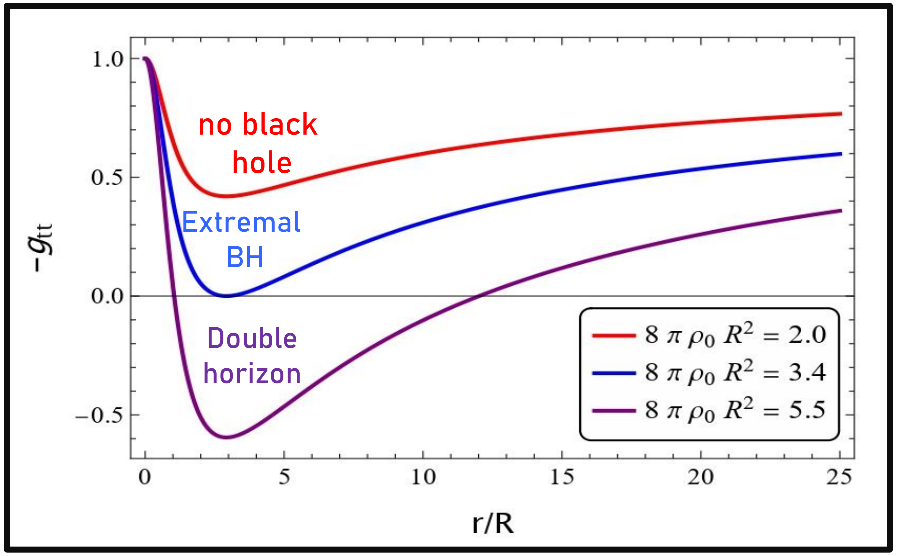
Combination of powers of $\frac{1}{r}$, positive powers of $\ln(r)$ → Polyhomogeneous spacetime

P. T. Chru´sciel, M. A. MacCallum, D. B. Singleton, Phil. Trans. R. Soc. A 350, 113–141(1995).

❑ Locations of Horizons:

Based on the number of *real positive roots* of the equation $f(r) = 0$, we have:

- ➢ Double horizon for $8\pi\rho_0 R^2 > 3.448$
- ➢ Single horizon for $8\pi\rho_0 R^2 = 3.448$
- ➢ Horizon-less when $8\pi\rho_0 R^2 < 3.448$



❑ Regularity of curvature invariants:

$$g_{\mu\nu}R^{\mu\nu} = \frac{8\pi\rho_0 R^3(r^2+4R^2)}{(r^2+R^2)^{5/2}} \qquad R_{\mu\nu}R^{\mu\nu} = \frac{32\pi^2\rho_0^2 R^6(5r^4+4r^2R^2+8R^4)}{(r^2+R^2)^5}$$

$$\lim_{r\to 0} R_{\mu\nu\lambda\delta}R^{\mu\nu\lambda\delta} = \frac{512\pi^2\rho_0^2}{3}$$

Metric is regular all over the radial coordinate.

❑ Energy conditions:

- The diagonal elements of the energy-momentum tensor are following:

$$\rho = -\tau = \rho_0\left(1 + \frac{r^2}{R^2}\right)^{-3/2}$$

$$p = \frac{\rho_0 R^3 (r^2 - 2R^2)}{2(r^2 + R^2)^{5/2}}$$

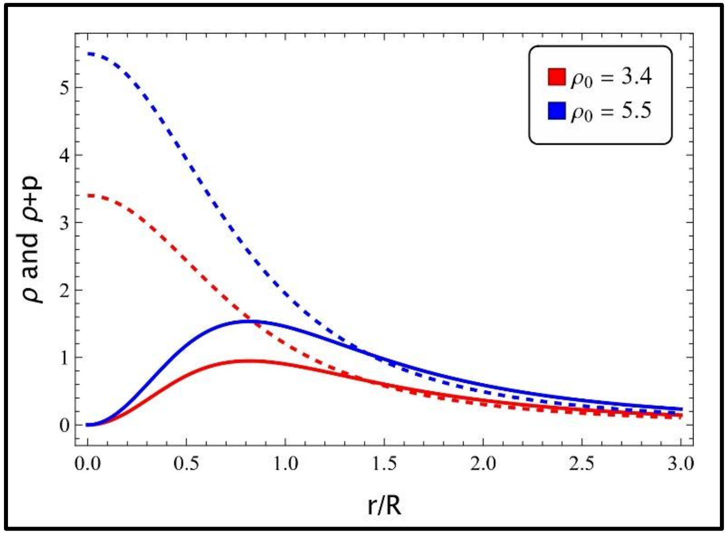


Fig: The dashed lines and solid lines represent ρ and ρ + p, respectively.

- Null Energy Condition (NEC), Weak Energy Condition (WEC):

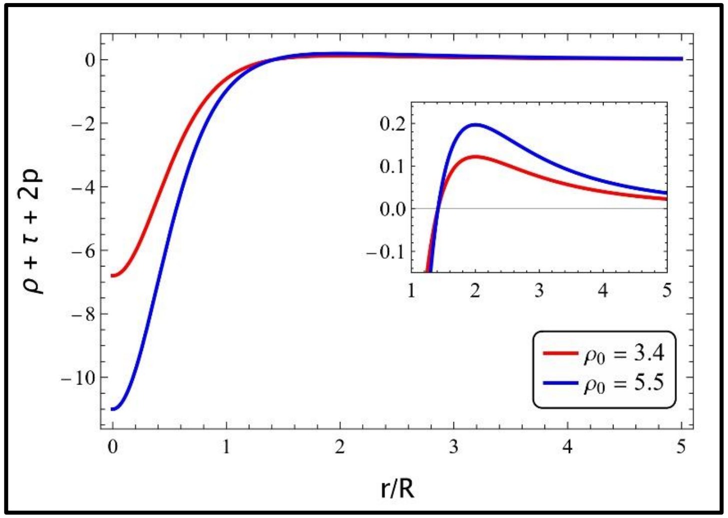
$$\rho > 0, \qquad \rho + \tau = 0, \qquad \rho + p = \frac{3\rho_0 R^3 r^2}{2(r^2 + R^2)^{5/2}} > 0$$

NEC and WEC hold for all r

- Strong Energy Condition (SEC):

$$\rho + \tau + 2p = \frac{\rho_0 R^3 (r^2 - 2R^2)}{(r^2 + R^2)^{5/2}}$$

SEC is violated for $r < \sqrt{2}R$



❑ Matter source for the geometry:

- In the GR coupled to matter scenario:

$$S = \int d^4x \sqrt{-g} \left(\frac{R}{\kappa} + L(F) \right)$$

- The matter Lagrangian:

$$L(F) = \frac{\delta (2F)^{3/4}}{(1 + \gamma \sqrt{2F})^{3/2}}$$

- Here, $F = \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$ $\delta = -\frac{\rho_0 R^3}{q_m^{3/2}}$ $\gamma = \frac{R^2}{q_m}$

- The nonzero component of Maxwell field strength tensor: $F_{\theta\phi} = -q_m \sin\theta$

- q_m is the total magnetic charge.

- $\delta = 0,\ L(F) = 0 \rightarrow$ Schwarzschild solution

The known regular black holes

- When $\nu = \alpha$ and $\mu \geq 3$, the density profile becomes:

$$\rho = \frac{\rho_0 \left(\frac{r}{R}\right)^{\mu-3}}{\left(1 + \left(\frac{r}{R}\right)^{\nu}\right)^{\frac{\mu+\nu}{\nu}}}$$

The corresponding metric function is *generalized Neves-Saa* regular solution:

$$f(r) = 1 - \frac{8\pi\rho_0 R^2 (r/R)^{\mu-1}}{\mu(1 + (r/R)^{\nu})^{\frac{\mu}{\nu}}}$$

J.C.S. Neves, A. Saa, PLB 734, 44 (2014).

For $\mu = 3, \nu = 2 \rightarrow$ Bardeen solution, $\mu = \nu = 3 \rightarrow$ Hayward solution

- Other solutions:

Parameters	Density profile	Metric function
$\mu = 3$ $\nu = 2$ $\alpha = 1$	$\rho(r) = \frac{\rho_0}{\left(1 + \frac{r^2}{R^2}\right)^2}$	$f(r) = 1 - 8\pi R^2 \rho_0 \frac{\tan^{-1}(r/R)}{2r/R} + \frac{8\pi\rho_0 R^4}{2(r^2 + R^2)}$ I. Dymnikova, CQG 21, 4417-4429 (2004).
$\mu = 3$ $\nu = 3$ $\alpha = 1$	$\rho = \frac{\rho_0}{\left(1 + \left(\frac{r}{R}\right)^3\right)^{\frac{4}{3}}}$	$f(r) = 1 - \frac{8\pi\rho_0 R^3}{r}\left\{1 - \left(1 + \left(\frac{r}{R}\right)^3\right)^{-\frac{1}{3}}\right\}$ K. A. Bronnikov, IJMPD 27, 1841005 (2018).

A regular defect solution

- We consider the *Pseudo-isothermal* dark matter profile $(\mu = 3, \nu = 2, \alpha = -1)$

$$\rho(r) = \frac{\rho_0 R^2}{r^2 + R^2}$$

K.G. Begeman, A.H. Broeils, R.H. Sanders, MNRS 249, 523 (1991).

- The corresponding *metric function*:

$$f(r) = 1 - 8\pi R^2 \rho_0 + 8\pi R^2 \rho_0 \frac{\tan^{-1}\left(\frac{r}{R}\right)}{\frac{r}{R}}$$

- For small r, metric function behaves like de-Sitter, $f(r) \approx 1 - \frac{8\pi\rho_0}{3} r^2$

- The asymptotic behaviour: $f(r) \approx 1 - 8\pi R^2 \rho_0$

The geometry is not asymptotically Minkowski ⇒ Solid angle deficit

- To understand the deficit, we perform the following transformation:

$$\tilde{r} = \frac{r}{\sqrt{1 - 8\pi R^2 \rho_0}}, \qquad \tilde{t} = t\sqrt{1 - 8\pi R^2 \rho_0}$$

- The transformation is allowed for, $\mathbf{8\pi R^2 \rho_0 < 1}$

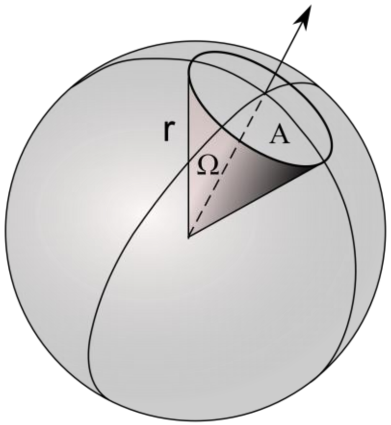
- The transformed metric:

$$ds^2 = -\tilde{f}(\tilde{r})d\tilde{t}^2 + \frac{d\tilde{r}^2}{\tilde{f}(\tilde{r})} + (1 - 8\pi R^2\rho_0)\tilde{r}^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

- Here, $\tilde{f}(\tilde{r})$ reaches *unity asymptotically*.

- Surface area of the spherical surface with radius $\tilde{r}$ is $4\pi\left(1 - 8\pi R^2\rho_0\right)\tilde{r}^2$.

Surface area is less than the entire sphere ⟶ deficit / defect

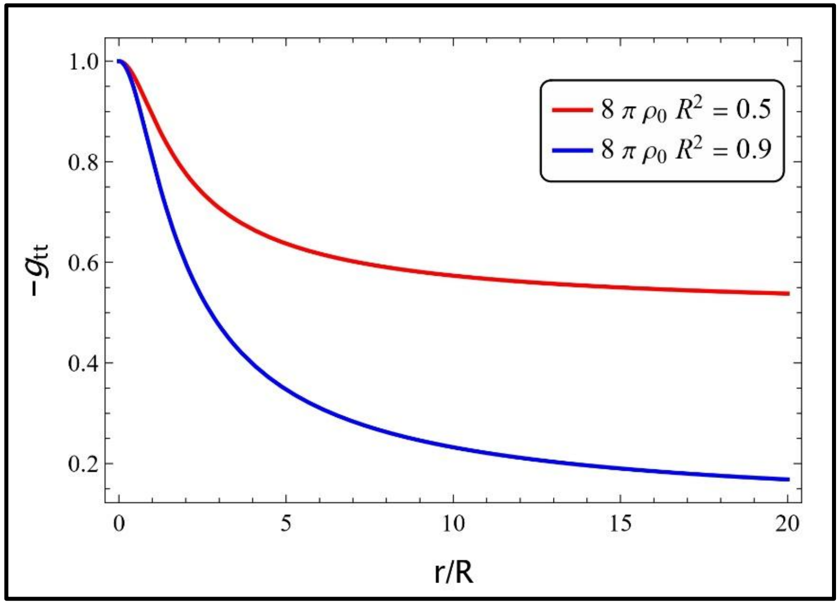


Source: Wikipedia

Spacetime structure:

- No real roots of the equation $f(r) = 0$.
- No horizon-like structure.

The geometry represents a regular, horizon-less, defect spacetime



❑ Regularity of curvature scalars:

$$g_{\mu\nu}R^{\mu\nu} = \frac{16\pi\rho_0 R^2(r^2+2R^2)}{(r^2+R^2)^2} \qquad R_{\mu\nu}R^{\mu\nu} = \frac{128\pi^2\rho_0^2 R^2(r^4+2r^2R^2+2R^4)}{(r^2+R^2)^4}$$

$$\lim_{r\to 0} R_{\mu\nu\lambda\delta}R^{\mu\nu\lambda\delta} = \frac{512\pi^2\rho_0^2}{3}$$

Metric is regular all over the radial coordinate.

❑ Embedding diagram:

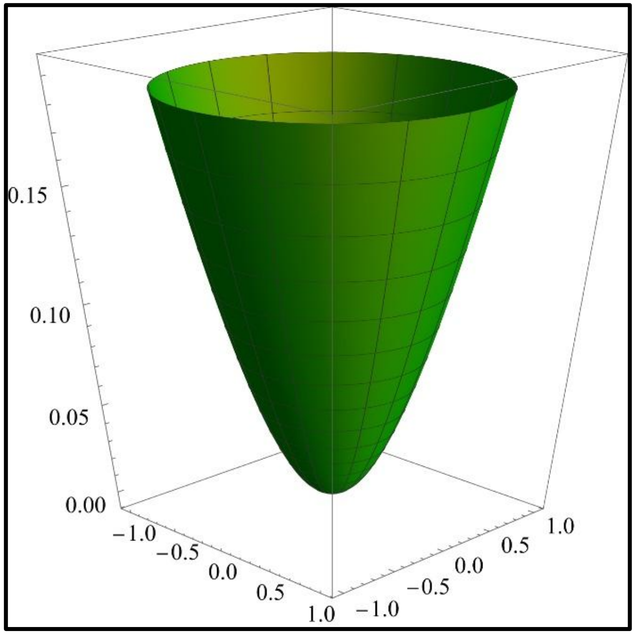
- Embedding the $t = constant$ and $\theta = \frac{\pi}{2}$ slice in the 3d Euclidean cylindrical geometry
- The 2d slice:

$$ds^2 = \frac{dr^2}{1 - 8\pi R^2\rho_0 + 8\pi R^2\rho_0 \frac{\tan^{-1}\left(\frac{r}{R}\right)}{\frac{r}{R}}} + r^2 d\phi^2$$

- Cylindrical geometry:

$$ds^2 = d\zeta^2 + \zeta^2 d\phi^2 + dz^2$$

- Profile function $z(r)$



Energy conditions:

- The diagonal elements of the energy-momentum tensor are following:

$$\rho = -\tau = \frac{\rho_0 R^2}{r^2 + R^2}$$

$$p = -\frac{\rho_0 R^4}{(r^2 + R^2)^2}$$

- Null Energy Condition (NEC) and Weak Energy Condition (WEC):

$$\rho > 0, \qquad \rho + \tau = 0, \qquad \rho + p = \frac{\rho_0 R^2 r^2}{(r^2 + R^2)} > 0$$

NEC and WEC hold for all r

- Strong Energy Condition (SEC):

$$\rho + \tau + 2p = -\frac{2\rho_0 R^4}{(r^2 + R^2)^2} < 0$$

SEC is violated for all r

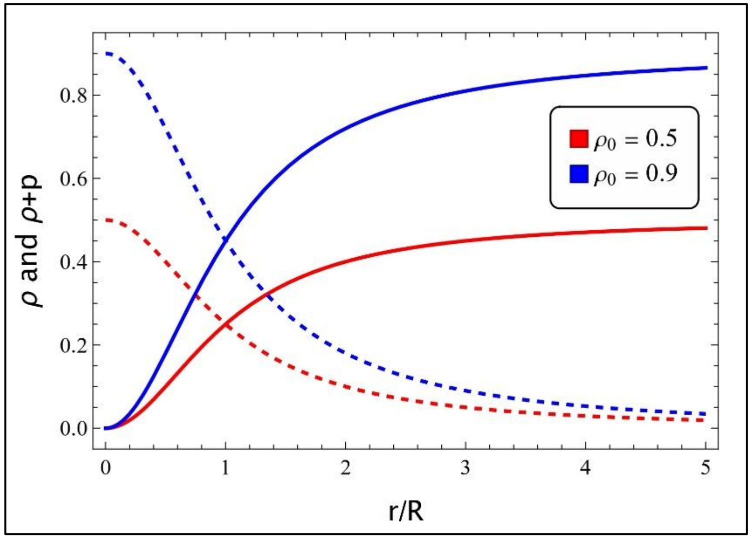
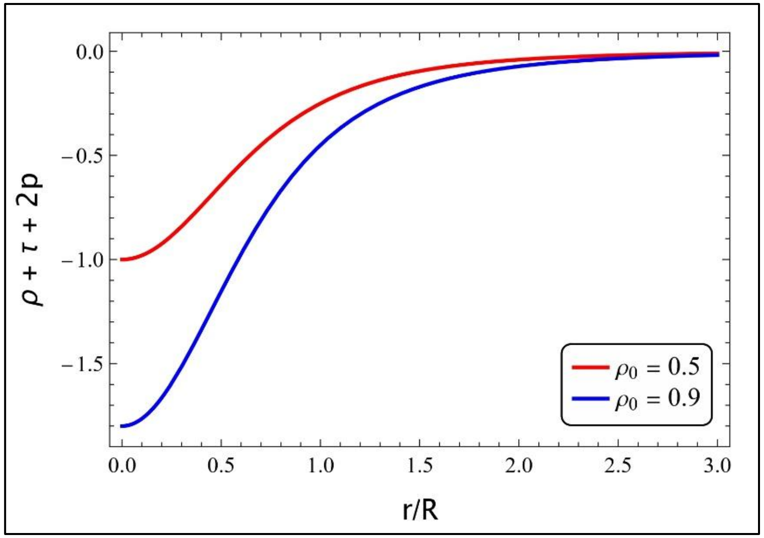


Fig: The dashed lines and solid lines represent ρ and ρ + p, respectively.



Lagrangian model for the required matter:

- The model of a cloud of strings is based on a surface bivector $\Sigma_{\mu\nu}$ that spans the 2D timelike world sheet of strings. P. S. Letelier, PRD 20, 1294 (1979).

$$\Sigma^{\mu\nu} = \epsilon^{AB} \frac{\partial x^{\mu}}{\partial \zeta^{A}} \frac{\partial x^{\nu}}{\partial \zeta^{B}}$$

- The energy-momentum tensor: $T^{\mu\nu} = \rho\sqrt{-h}\frac{\Sigma^{\mu\lambda}\Sigma^{\nu}_{\lambda}}{(-h)}$

- For the fluid of strings, the energy-momentum tensor:

$$T^{\mu\nu} = \left(p + \rho\sqrt{-h}\right) \frac{\Sigma^{\mu\lambda}\Sigma^{\nu}{}_{\lambda}}{(-h)} + p g^{\mu\nu}$$

P. S. Letelier, Nuov. Cim. B 63, 519–528 (1981).

- For the symmetries of the defect metric, the diagonal elements:

$$T^{\mu}{}_{\nu} = [-\rho(r), -\rho(r), p, p]$$

- The components are: $\rho = \frac{\rho_0 R^2}{r^2 + R^2}, \qquad p = -\frac{\rho_0 R^4}{(r^2 + R^2)^2}$

- Equation of state: $p = -\frac{1}{\rho_0}\rho^2$ Polytropic fluid of strings

- The asymptotic expansion of ρ and p are,

$$\rho \approx \frac{\rho_0 R^2}{r^2}, \quad \text{and} \quad p \to 0$$

- The corresponding metric function is $f(r) \approx 1 - 8\pi R^2 \rho_0$

It can be associated with the cloud of strings

Asymptotically, the geometry represents a flat spacetime surrounded by a cloud of strings, which may be a reason behind the appearance of the solid angle deficit.

Geodesic completeness

- Radial time-like geodesic: $\dot{r}^2 = E^2 - V_{eff}$
- Conserved quantity; $E = -g_{tt}\dot{t}$
- Affine parameter: $\lambda(r_1, r_2) = \int_{r_1}^{r_2} \frac{dr}{\sqrt{\dot{r}^2}}$

geodesic completeness ⇨ $\lambda(-\infty, +\infty)$

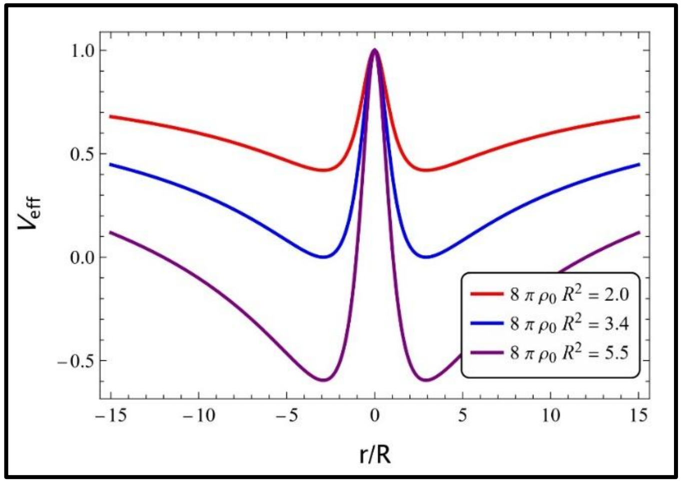
$r = 0$	$\dot{r}^2$ at $r = 0$	$\lambda(R, 0)$	Extension of spacetime	$\dot{r}^2$ at negative 'r'
singular	diverge	finite	Not possible	😐
regular	finite	finite	Possible (in *-ve values* of 'r')	i) Diverges ☒ ii) Smooth and continuous ☑

Everywhere finite behaviour of $-g_{tt}$ ⇨ a Geodesically complete spacetime

T. Zhou and L. Modesto, PRD 107, 044016 (2023)

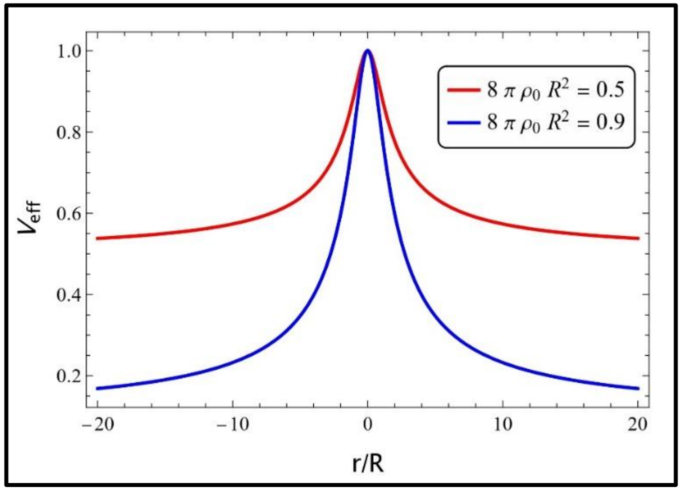
Geodesic completeness of the regular black hole:

$$V_{eff} = -g_{tt} = 1 + \frac{8\pi\rho_0 R^3}{\sqrt{r^2+R^2}} + \frac{8\pi\rho_0 R^3}{r}\ln\left(\frac{\sqrt{r^2+R^2}-r}{R}\right)$$



Geodesic completeness of defect solution:

$$V_{eff} = -g_{tt} = 1 - 8\pi R^2\rho_0 + 8\pi R^2\rho_0 \frac{\tan^{-1}\left(\frac{r}{R}\right)}{\frac{r}{R}}$$



Thus, both of the solutions are geodesically complete.

An alternative approach

- Anisotropic TOV equation:

$$\frac{d\tau}{dr} = -\frac{(\rho+\tau)(m(r)+4\pi r^3\tau)}{r(r-2m(r))} + \frac{2}{r}(p-\tau)$$

- We consider the equation of state:

$$\tau = -\rho, \qquad p = a\rho + \frac{b}{\rho_0^{\lambda-1}}\rho^{\lambda}$$

- Solution of the TOV equation:

$$\rho = \rho_0 \left(-\frac{(1+a)/b}{1+\left(\frac{r}{R}\right)^{2(1+a)(\lambda-1)}} \right)^{\frac{1}{\lambda-1}}$$

- Physicality conditions: $a+1>0,\ \lambda>0$ and $b<0$

- $a=\frac{1}{2}, b=-\frac{3}{2}, \lambda=\frac{5}{3} \rightarrow$ King density profile $\rightarrow$ The new regular black hole

- $a=0, b=-1, \lambda=2 \rightarrow$ Pseudo-isothermal density profile $\rightarrow$ The defect solution

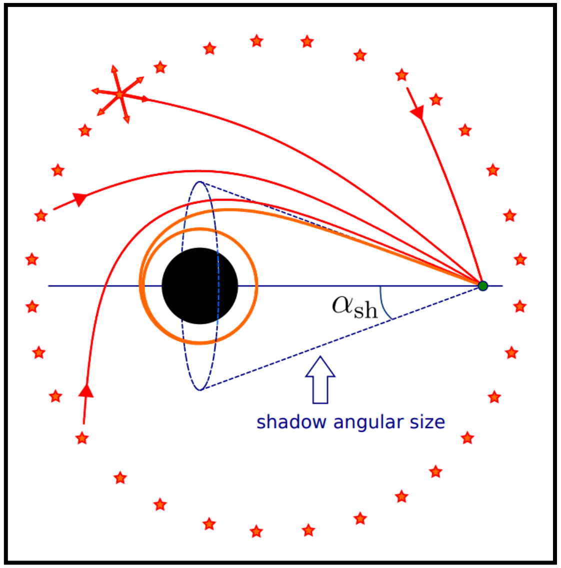
Black hole shadow

○ **Simple shadow**

Source:tuntex-carpet

○ **Silhouette**

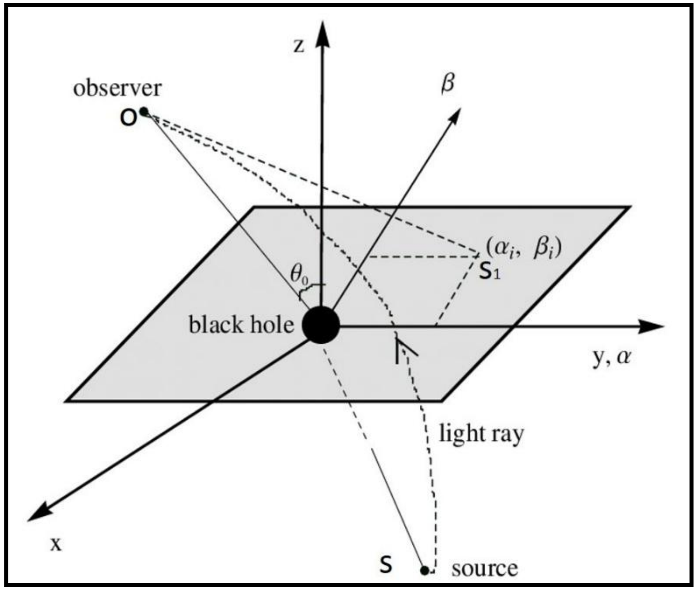
Source:quora



Source :V. Perlick and O. Y. Tsupko, Phys. Rep. 947, 1-39 (2022)

Shadow radius:

$$r_{sh}^2 = \frac{r_{ph}^2}{-g_{tt}(r_{ph})}$$



Source :researchgate

Constraints on metric parameters from EHT

- The observed angular diameter for M87* is $42 \pm 3\ \mu as$, distance from observer $(16.8 \pm 0.8)\ Mpc$ Astrophys. J. Lett. 875, L1 (2019), Astrophys. J. Lett. 694, 556–572 (2009)

$$\rho_0 \sim 10^2\ kg/m^3 \text{ and } R \sim 10^{12}\ meter$$

- For Sgr A*, the angular diameter is $51.8 \pm 2.3\ \mu as$, distance measurement is (8277 ± 9 ± 33)pc Astrophys. J. Lett. 930, L12 (2022), Astron. Astrophys. 657, L12 (2022).

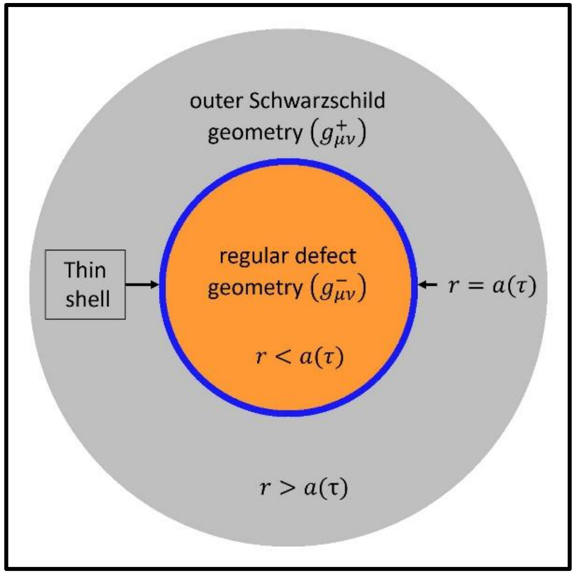
$$\rho_0 \sim 10^8\ kg/m^3 \text{ and } R \sim 10^9\ meter$$

Model of a stable star (Gravastar)

- The defect geometry have following features:
  - It behaves like de-Sitter space at the centre.
  - Its embedding diagram illustrates that its geometry is like the interior of a star.
  - The polytropic fluid of strings can model the required matter
- We consider the Visser–Wiltshire's dynamically stable thin shell model.

M. Visser, D.L. Wiltshire, CQG 21, 1135 (2004).

- The three-layer model is following:
  - An outer Schwarzschild geometry.
  - A thin shell with *surface density, surface tension*.
  - Interior regular defect geometry.



- Sen-Israel-Darmois junction condition:

W. Israel, NCB 48, 463 (1967).

  - Induced metric on shell is same from inside and outside.
  - Jump in the extrinsic curvature is proportional to the surface energy-momentum.

Null surface stress-energy → boundary. Finite stress energy → thin shell.

- The exterior and interior metrics:

$$ds_\pm^2 = -f_\pm(r)dt^2 + \frac{dr^2}{f_\pm(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \qquad f_\pm(r) = 1 - \frac{2m_\pm(r)}{r}$$

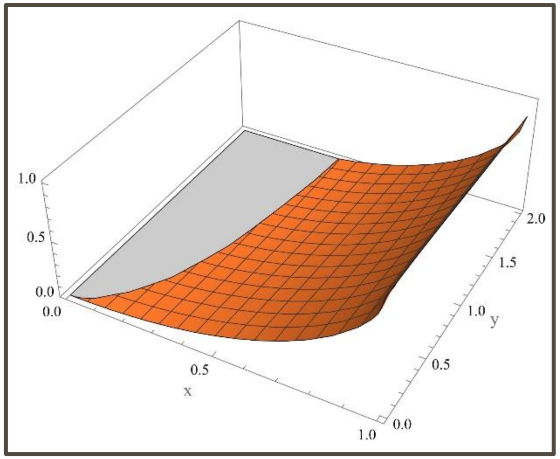
$$m_+(a) = M, \qquad m_-(a) = 4\pi R^2\rho_0 a - 4\pi R^3\rho_0 \tan^{-1}(a/R)$$

- We consider the junction surface at $r = a_0,\ a_0 > 2M$

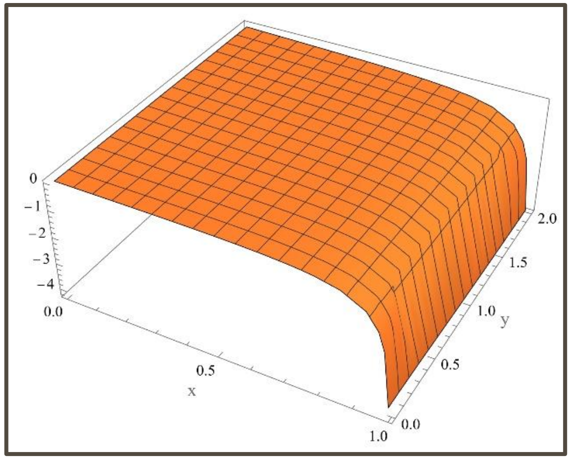
- The new variables:

$$\mu(a_0) = 8\pi M\sigma(a_0), \quad x = 2M/a_0, \quad y = 2M/R \text{ and } \Pi(a_0) = 16\pi M v(a_0)$$

- The ratio of $\mu(a_0)$ and $\Pi(a_0)$ is defined as equation of state parameter for the matter in the shell.

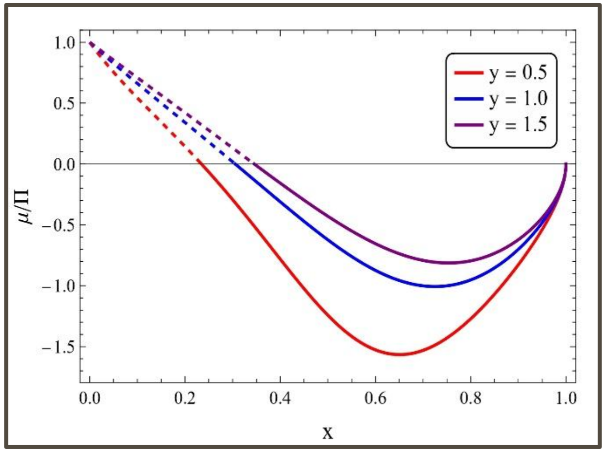


Surface energy density $\mu(a_0)$



Surface tension $\Pi(a_0)$

$8\pi R^2\rho_0 = 0.5$



Equation of state

Summary and conclusion

- The parametrized Dekel-Zhao density profile is used to construct new regular solutions.
- We construct a new regular black hole solution which is sourced by NLE Lagrangian.
- When the density profile is pseudo-isothermal, the geometry represents a regular defect solution.
- We construct the solutions using TOV equation.
- Finally, we discuss the astrophysical applications.

THANK YOU

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A brief of Visser-Wiltshire's model

- Given the interior and exterior metrics:

$$ds_{\pm}^2 = -f_{\pm}(r)dt^2 + \frac{dr^2}{f_{\pm}(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \qquad f_{\pm}(r) = 1 - \frac{2m_{\pm}(r)}{r}$$

- Connected along a dynamical time-like hyper-surface (Σ) at $r = a(t)$,
- Sen-Israel-Darmois junction condition: W. Israel, Nuovo Cimento B (1965-1970) 48, 463 (1967).
  - Induced metric on Σ is same from inside and outside.
  - The jump in the extrinsic curvature is directly proportional to the surface energy-momentum tensor S_{ij} at the shell.
- The extrinsic curvature: $K_{ij} = \nabla_\nu n_\mu e^\mu_{(i)} e^\nu_{(j)}$
- The surface energy-momentum tensor: $[[K_{ij} - K g_{ij}]] = -8\pi S_{ij}$
- Surface energy density: $\sigma = -\frac{1}{4\pi}(K_\theta^{\theta +} - K_\theta^{\theta -})$
- Surface tension: $v = -\frac{1}{8\pi}(K_\tau^{\tau +} + K_\theta^{\theta +} - K_\tau^{\tau -} - K_\theta^{\theta -})$

Vanishing surface stress-energy makes junction surface as boundary. Finite stress energy makes junction thin shell.

Junction formalism:

$$g_{ij} d\xi^i d\xi^j = -d\tau^2 + a^2(\tau)(d\theta^2 + \sin^2\theta d\phi^2)$$

$$n_\mu^\pm = \left(-\dot{a}, \frac{\sqrt{f_\pm(a) + \dot{a}^2}}{f_\pm(a)}, 0, 0\right) \qquad U_\pm^\mu = \left(\frac{\sqrt{f_\pm(a) + \dot{a}^2}}{f_\pm(a)}, \dot{a}, 0, 0\right)$$

$$K_\tau^{\tau\pm} = \frac{\ddot{a} + f'_\pm(a)/2}{\sqrt{f_\pm(a) + \dot{a}^2}}$$

$$K_\theta^{\theta\pm} = K_\phi^{\phi\pm} = \frac{1}{a}\sqrt{f_\pm(a) + \dot{a}^2}$$

$$\frac{d}{d\tau}(\sigma a^2) = v\frac{d}{d\tau}(a^2)$$